

Low energy physics and left-right symmetry. Bounds on the model parameters.

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Abstract

In any gauge model with spontaneous symmetry breakdown the gauge boson masses and their mixings are not independent quantities. They are interconnected through the vacuum expectation values (VEVs) of the Higgs sector. We discuss the low-energy experiments, namely electron-hadron, neutrino-hadron and neutrino-electron processes in the frame of the Manifest Left-Right Symmetric model and show the impact of these dependencies on the possible heavy gauge boson masses and mixings. At 90 % C.L., we obtain $M_{Z_2} \geq 1475$ GeV for phenomenologically favorable models, without $W_L - W_R$ mixing and $M_{Z_2} \geq 1205$ GeV in the other extreme case when a maximal mixing is possible. If we consider the left-right symmetric model parameters without any constraints from the Higgs sector these limits get down to 410 GeV. Bounds on the Z_2 mixing angle as well as the W_2 mass and its mixing angle are also given.

1 Introduction

Undoubtedly, we can say that this decade is a further, permanent progress in experimental high energy particle physics. Let's modestly mention LEP achievements [1] as well as top discovery [2]. However, not less impressive results in low energy physics have been obtained. Especially much has been done in neutral current physics, where data coming from both deep inelastic neutrino-hadron, neutrino-electron scattering as well as electron-hadron interactions have been enriched lately by exquisitely precise measurements of parity nonconservation (PNC) in heavy atoms, such as cesium [3] and thalium [4]. The CCFR collaboration data on quark-Z boson couplings has also improved [5]. This kind of experiments is a nice (and not too expensive regarding high energy collider physics) tool to probe the standard electroweak model (SM) and its parameters. Moreover, searches beyond the standard model physics using low-energy data complement quite well the efforts made at high energy

colliders. To visualize this statement we use in this work, as a representative for ‘new physics’, the classical Manifest Left-Right Symmetric (MLRS) model. Its principle advantage over the SM is space inversion invariance at high energies, implied not only by the gauge group but also by a discrete symmetry (replacement of the left by the right fields and *vice versa*). As a consequence the left and right couplings g_L , g_R are equal, $g_L = g_R = g$, and the Yukawa matrices in the quark and lepton sectors are hermitian. A minimal Higgs sector with a bidoublet Φ , and left Δ_L and right Δ_R triplets is adopted [6,7], with the additional assumption that only Φ and Δ_R have non-vanishing VEVs¹. In this model we have four non-standard parameters, namely, additional gauge boson masses M_{W_2} , M_{Z_2} and mixing angles in both charged and neutral gauge sectors (ζ , ϕ). We use them to parametrize the mentioned low-energy neutral data. These parameters are not independent of each other, as they are functions of the VEVs of Φ and Δ_R . Based on this fact we end up with two independent factors $\gamma = M_{Z_1}^2/M_{Z_2}^2$ and $\epsilon = 2\kappa_1\kappa_2/(\kappa_1^2 + \kappa_2^2)$. This way, exploiting the full strength of the MLRS enabled us to obtain quite impressive limits on the M_{Z_2} mass (much above 1 TeV). The results are better than those from direct searches at high energy hadron colliders and comparable to those extracted from the LEPI data. All numerics are done with the CERN code MINUIT [8].

2 M_{Z_2} mass and low-energy neutral current experiments

The low energy processes’ momentum transfer being much smaller than the intermediate gauge boson masses, contact four-fermion Lagrangians can be effectively used. Four-fermion neutrino-hadron (νN), neutrino-electron (νe) and parity-violating electron-hadron (eN) interactions can be written in the conventional form as follows [9]

$$L^{\nu N} = -\frac{G_F}{\sqrt{2}} \bar{\nu} \gamma^\mu (1 - \gamma_5) \nu \sum_{i=u,d} [\epsilon_L(i) \bar{q}_i \gamma_\mu (1 - \gamma_5) q_i + \epsilon_R(i) \bar{q}_i \gamma_\mu (1 + \gamma_5) q_i], \quad (1)$$

$$L^{\nu e} = -\frac{G_F}{\sqrt{2}} \bar{\nu} \gamma^\mu (1 - \gamma_5) \nu \bar{e}_i \gamma_\mu (g_V^{\nu e} - g_A^{\nu e} \gamma_5) e, \quad (2)$$

$$L^{\nu N} = \frac{G_F}{\sqrt{2}} \sum_{i=u,d} [C_{1i} \bar{e} \gamma_\mu \gamma_5 e \bar{q}_i \gamma_\mu q_i + C_{2i} \bar{e} \gamma_\mu e \bar{q}_i \gamma_\mu \gamma_5 q_i]. \quad (3)$$

Here, $\epsilon_{L,R}(i)$, $g_{V,A}^{\nu e}$, C_{ij} are model-dependent coefficients. It is usual to consider

¹

$$\langle \Phi \rangle = \frac{1}{\sqrt{2}} \begin{pmatrix} \kappa_1 & 0 \\ 0 & \kappa_2 \end{pmatrix}, \quad \langle \Delta_R \rangle = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 & 0 \\ v_R & 0 \end{pmatrix}, \quad \langle \Delta_L \rangle = 0.$$

only pure left-handed currents in $L^{\nu e}$ and $L^{\nu N}$. In the SM they can be derived by comparison with the neutral current Lagrangian (T_{3i}^L and Q_i are the weak isospin of fermion i and its charge, respectively)

$$L_{NC}^{SM} = \frac{g}{2 \cos \Theta_W} \sum_{u,d,\nu,e} \bar{\Psi}_i \gamma^\mu (g_V^i - g_A^i \gamma_5) \Psi_i Z_\mu \quad (4)$$

with

$$g_V^i \equiv T_{3i}^L - 2Q_i \sin^2 \Theta_W, \quad (5)$$

$$g_A^i \equiv T_{3i}^L. \quad (6)$$

Using The SM definition $\frac{G_F}{\sqrt{2}} = \frac{\pi\alpha}{2 \sin^2 \Theta_W (1-\Delta r) M_{W_1}^2}$, we get

$$\epsilon_L^{SM}(i) = \rho_{\nu N} (T_{3i} - Q_i \kappa_{\nu N} \sin^2 \Theta_W) + \lambda_{iL}, \quad (7)$$

$$\epsilon_R^{SM}(i) = \rho_{\nu N} (-Q_i \kappa_{\nu N} \sin^2 \Theta_W) + \lambda_{iR}, \quad (8)$$

$$C_{1i}^{SM} = \rho'_{eq} (-T_{3i} + 2Q_i \kappa'_{eq} \sin^2 \Theta_W) + \lambda_{1i}, \quad (9)$$

$$C_{2i}^{SM} = \rho_{eq} (-1/2 + 2\kappa_{eq} \sin^2 \Theta_W) (2T_{3i}) + \lambda_{2i}, \quad (10)$$

$$(g_V^{\nu e})^{SM} = \rho_{\nu e} (-1/2 + \kappa_{\nu e} \sin^2 \Theta_W), \quad (11)$$

$$(g_A^{\nu e})^{SM} = \rho_{\nu e} (-1/2). \quad (12)$$

The ρ , λ and κ factors include the effects of one-loop radiative corrections to the low energy processes [9]. At tree level $\rho = \kappa = 1$ and $\lambda = 0$.

Now let us proceed to the left-right symmetric model.

The masses of the gauge bosons ($M_{Z_{1,2}}, M_{W_{1,2}}$) and the mixing angles ζ, ϕ in charged and neutral gauge sectors are the following ($\kappa_+ = \sqrt{\kappa_1^2 + \kappa_2^2}$) [10,11]

$$M_{W_{1,2}}^2 = \frac{g^2}{4} \left[\kappa_+^2 + v_R^2 \mp \sqrt{v_R^4 + 4\kappa_1^2 \kappa_2^2} \right], \quad (13)$$

$$M_{Z_{1,2}}^2 = \frac{1}{4} \left\{ \left[g^2 \kappa_+^2 + 2v_R^2 (g^2 + g'^2) \right] \mp \sqrt{[g^2 \kappa_+^2 + 2v_R^2 (g^2 + g'^2)]^2 - 4g^2 (g^2 + 2g'^2) \kappa_+^2 v_R^2} \right\}, \quad (14)$$

$$\tan 2\xi = -\frac{2\kappa_1 \kappa_2}{v_R^2}, \quad (15)$$

$$\sin 2\phi = -\frac{g^2 \kappa_+^2 \sqrt{\cos 2\Theta_W}}{2 \cos^2 \Theta_W (M_{Z_2}^2 - M_{Z_1}^2)}. \quad (16)$$

Obviously these are functions of the three VEVs v_R, κ_1, κ_2 . Certainly, $M_{W(Z)_1} < M_{W(Z)_2}$, so $\kappa_+^2, \kappa_1 \kappa_2 \ll v_R^2$ and we expand the above formulas leaving terms up to $O\left(\left(\frac{\kappa_+}{v_R}\right)^2, \left(\frac{\kappa_1 \kappa_2}{v_R^2}\right)\right)$ (we will comment on this approximation at the end of the Chapter).

The result can be cast in the form $\left(\beta = \frac{M_{W_1}^2}{M_{W_2}^2}\right)$ [10–12]

$$\gamma \equiv \frac{M_{Z_1}^2}{M_{Z_2}^2} = \frac{\cos 2\Theta_W}{2 \cos^4 \Theta_W} \beta, \quad (17)$$

$$\zeta = -\epsilon \beta, \quad (18)$$

$$\phi = -\frac{(\cos 2\Theta_W)^{3/2}}{2 \cos^4 \Theta_W} \beta, \quad (19)$$

and

$$\rho_{LR} \equiv \frac{M_{W_1}^2}{M_{Z_1}^2 \cos^2 \Theta_W} = 1 + \left[-\epsilon^2 + \frac{1}{2}(1 - \tan^2 \Theta_W)^2\right] \beta, \quad (20)$$

where

$$\epsilon = \frac{2\kappa_1 \kappa_2}{\kappa_1^2 + \kappa_2^2}, \quad 0 \leq \epsilon \leq 1. \quad (21)$$

In the MLRS model the neutral current interaction for any fermions can be written

$$L_{NC} = \frac{e}{2 \sin \Theta_W \cos \Theta_W} \sum_{i=up, down, l, \nu} \sum_{j=1,2} \bar{\psi}_i \gamma^\mu \left[A_L^{ji} \Omega_L^i P_L + A_R^{ji} \Omega_R^i P_R \right] \psi_i Z_{j\mu}. \quad (22)$$

The couplings $A_{L,R}^{1,2;i}$ are given by

$$A_L^{1i} = \cos \phi g_L^i + \sin \phi g_L'^i, \quad (23)$$

$$A_R^{1i} = \cos \phi g_R^i + \sin \phi g_R'^i, \quad (24)$$

$$A_L^{2i} = \sin \phi g_L^i - \cos \phi g_L'^i, \quad (25)$$

$$A_R^{2i} = \sin \phi g_R^i - \cos \phi g_R'^i, \quad (26)$$

where

$$g_L^i = 2T_{3i}^L - 2Q_i \sin^2 \Theta_W, \quad (27)$$

$$g_L^i = \frac{2 \sin^2 \Theta_W}{\sqrt{\cos 2\Theta_W}} (Q_i - T_{3i}^L), \quad (28)$$

$$g_R^i = -2Q_i \sin^2 \Theta_W, \quad (29)$$

$$g_R^i = \frac{2}{\sqrt{\cos 2\Theta_W}} \left(Q_i \sin^2 \Theta_W - T_{3i}^R \cos^2 \Theta_W \right). \quad (30)$$

$\Omega_{L,R}$ are analogous to Cabbibo-Kobayashi mixing matrices in the charged sector and are the identity matrices for charged fermions

$$\Omega_{L,R}^i = I \quad \text{for } i = u, d, l.$$

To have a link with the model independent Lagrangians (Eq.(1-3)) we now assume that only pure left-handed neutrinos play a role in neutral low energy physics². Then $\Omega_L^\nu \simeq I, \Omega_R^\nu \simeq 0$.

We can now, quite analogously to the SM case, find low energy LR model coefficients

$$\epsilon_{L,R}^{LR}(i) = \Lambda(A_L^{1\nu} A_{L,R}^{1i} + \gamma A_L^{2\nu} A_{L,R}^{2i}), \quad (31)$$

$$C_{1i}^{LR} = \Lambda(g_A^{1l} g_V^{1i} + \gamma g_A^{2l} g_V^{2i}), \quad (32)$$

$$C_{2i}^{LR} = \Lambda(g_V^{1l} g_A^{1i} + \gamma g_V^{2l} g_A^{2i}), \quad (33)$$

$$(g_V^{\nu e})^{LR} = \Lambda(A_L^{1\nu} g_V^{1l} + \gamma A_L^{2\nu} g_V^{2l}), \quad (34)$$

$$(g_A^{\nu e})^{LR} = \Lambda(A_L^{1\nu} g_A^{1l} + \gamma A_L^{2\nu} g_A^{2l}), \quad (35)$$

where

$$g_{V,A}^i = \frac{1}{2} (A_L^{i\nu} \pm A_R^{i\nu}), \quad (36)$$

$$\Lambda = \frac{\rho_{LR}}{(\cos^2 \zeta + \beta \sin^2 \zeta)}. \quad (37)$$

The Λ factor is connected with the L-R definition of the G_F constant. If we assume the only natural situation when right-handed neutrinos are too heavy to be directly produced in the muon decay and the light are left-handed (negligible right-handed admixture) then the G_F definition follows [14]

$$\frac{G_F}{\sqrt{2}} = \frac{\pi\alpha}{2 \sin^2 \Theta_W M_{W_1}^2 (1 - \Delta r)} (\cos^2 \zeta + \beta \sin^2 \zeta). \quad (38)$$

The parameters of Eqs. (31-35) are 'bare', reproducing the 'bare' SM couplings for $\phi = \gamma = 0$. We improve them by adding the SM corrections (Eqs.(7)-(12)), so the data is fitted with

² If it was not true then we would certainly have had an indication on the long standing Dirac-Majorana neutrino nature problem [13].

$$\epsilon_{L,R}^{LR}(i) = \Lambda \left[A_L^{1\nu} \left(\cos \phi \epsilon_{L,R}^{SM}(i) + \frac{1}{2} \sin \phi g_{L,R}^i \right) + \frac{1}{2} \gamma A_L^{2\nu} A_{L,R}^{2i} \right], \quad (39)$$

$$C_{1i}^{LR} = \Lambda \left[\cos 2\phi - \sin 2\phi \frac{\sin^2 \Theta_W}{\sqrt{\cos 2\Theta_W}} \right] \left[C_{1i}^{SM} - \gamma (-T_{3i} + 2Q_i \sin^2 \Theta_W) \right] \quad (40)$$

$$C_{2i}^{LR} = \Lambda \left[\cos 2\phi - \sin 2\phi \frac{\sin^2 \Theta_W}{\sqrt{\cos 2\Theta_W}} \right] \left[C_{2i}^{SM} - \gamma \left(-\frac{1}{2} + 2 \sin^2 \Theta_W \right) (2T_{3i}) \right] \quad (41)$$

$$(g_V^{\nu e})^{LR} = \Lambda \left[A_L^{1\nu} \left(\cos \phi - \frac{\sin \phi}{\sqrt{\cos 2\Theta_W}} \right) (g_V^{\nu e})^{SM} + \gamma A_L^{2\nu} \left(\sin \phi + \frac{\cos \phi}{\sqrt{\cos 2\Theta_W}} \right) \left(-\frac{1}{2} + 2 \sin^2 \Theta_W \right) \right], \quad (42)$$

$$(g_A^{\nu e})^{LR} = \Lambda \left[A_L^{1\nu} (\cos \phi + \sin \phi \sqrt{\cos 2\Theta_W}) (g_A^{\nu e})^{SM} + \gamma A_L^{2\nu} \left(\sin \phi - \cos \phi \sqrt{\cos 2\Theta_W} \right) \left(-\frac{1}{2} \right) \right]. \quad (43)$$

In Table 1 we show the 1998 data [9] for all the couplings that are used. As the Standard Model one-loop corrections to these theoretical formulas (Eqs.(7)-(12)) have been calculated in the \overline{MS} scheme, we take the value of $\sin^2 \Theta_W$ in the same scheme $\sin^2 \Theta_W \equiv \hat{s}_Z^2 = 0.23124 \pm 0.00017$ [9]. In Fig.1 we show the 90 % C.L. allowed region for $\gamma - \phi$ parameters. The dotted line shows the results for the data from Table 1. The solid line follows from supplementing the previous with an additional parameter which measures the neutral to charged current cross section ratio in neutrino scattering off nuclei and is given by the CCFR collaboration [5]. This parameter in the frame of our model should be defined in the following way

$$\kappa^2 = 1.7897g_L^2 + 1.1479g_R^2 - 0.0916\delta_L^2 - 0.0782\delta_R^2 \quad (44)$$

where

$$g_{L,R}^2 = \left(\epsilon_{L,R}^{LR}(u) \right)^2 + \left(\epsilon_{L,R}^{LR}(d) \right)^2, \quad (45)$$

$$\delta_{L,R}^2 = \left(\epsilon_{L,R}^{LR}(u) \right)^2 - \left(\epsilon_{L,R}^{LR}(d) \right)^2. \quad (46)$$

The CCFR collaboration has found it to be

$$\kappa^2 = 0.5820 \pm 0.0041. \quad (47)$$

We can see that κ^2 does not change the predictions for the γ parameter. Using Eq.(17) we get $M_{Z_2} \geq 410$ GeV. This result is not substantially different from other analyses [10,15].

	Experimental Value	Correlations
$\epsilon_L(u)$	0.328 ± 0.0016	
$\epsilon_L(d)$	-0.440 ± 0.011	non-
$\epsilon_R(u)$	-0.179 ± 0.0013	Gaussian
$\epsilon_R(d)$	$-0.027^{+0.077}_{-0.048}$	
g_L^2	0.3009 ± 0.0028	
g_R^2	0.0328 ± 0.003	
Θ_L	2.50 ± 0.035	small
Θ_R	$4.56^{+0.42}_{-0.27}$	
$g_V^{\nu e}$	-0.041 ± 0.015	-0.04
$g_A^{\nu e}$	-0.507 ± 0.014	
C_{1u}	-0.216 ± 0.046	$-0.997 \quad -0.78$
C_{1d}	0.361 ± 0.041	0.78
$C_{2u} - \frac{1}{2}C_{2d}$	-0.03 ± 0.12	

Table 1

Data used for the neutral data analysis [9]. Appropriate formulas are given in the text. $g_{L,R}^2 = \epsilon_{L,R}^2(u) - \epsilon_{L,R}^2(d)$, $\tan \Theta_{L,R} = \frac{\epsilon_{L,R}(u)}{\epsilon_{L,R}(d)}$

Until now the left-right observables $\beta, \gamma, \zeta, \phi$ have been treated as independent, i.e. we do not take into account the relations (13)-(16) which reflect the fact that the VEVs link them to one another. When we use these relations the situation changes substantially (Fig2). We have chosen as independent two phenomenologically handful parameters: ϵ and γ . Different ϵ 's describe left-right models with different bidoublet VEVs κ_1, κ_2 . We know from phenomenological considerations that the reduction of FCNC favors left-right models with $\epsilon \simeq 0$ [7] (ellipses denoted with (b)). Then (see Eq.(18)) there is no $W_L - W_R$ mixing. However, to make this possibility open we also show the results for left-right models with $\epsilon \simeq 1$ (ellipses denoted with (a)). The dotted ellipses are obtained when all of the data from Table 1 is taken into account. The solid ones correspond to the inclusion of the κ^2 Eq.(47) parameter. Fig.2 shows that the MLRS relations among the fitted parameters $\beta, \gamma, \phi, \zeta$, and the CCFR data make it possible to shrink considerably the allowed space for the γ factor. From Eqs. (17)-(20) it is possible to find limits on the rest of the left-right parameters:

$$\left. \begin{array}{l} M_{Z_2} \geq 1475 \text{ GeV} \\ M_{W_2} \geq 875 \text{ GeV} \\ |\phi| \leq 0.0028 \text{ rad} \end{array} \right\} \text{ for models without } W_L - W_R \text{ mixing,}$$

$$\left. \begin{array}{l} M_{Z_2} \geq 1205 \text{ GeV} \\ M_{W_2} \geq 715 \text{ GeV} \\ |\zeta| \leq 0.013 \text{ rad} \\ |\phi| \leq 0.0042 \text{ rad} \end{array} \right\} \text{ for models with possible } W_L - W_R \text{ mixing } (\epsilon \neq 0)$$

These results are comparable with previous LEPI analyses ($M_{Z_2} \geq 0.8 \div 1.5$ TeV) [16] and better than that which follow from direct searches for additional gauge bosons in hadron colliders ($M_{Z_2} \geq 630$ GeV) [17]. Finally, let us comment on the approximation made in Eqs.(17)-(20). Our fitted observables (Eq.(36)-(41)) are functions of β, γ (so M_{W_2}, M_{Z_2}) and mixing angles ζ, ϕ . Taking into account exact formulas (13)-(16) for these quantities we have checked that, at 90 % C.L., quantities of the form $(\kappa_+/v_R)^2$ and $\kappa_1\kappa_2/v_R^2$ do not exceed at the worst case 0.04 so, when neglecting squares of them, relations (17)-(20) are quite reliable.

3 Conclusions

We point out the importance of examining non-standard models using relations among physical parameters such as masses and mixings that follow from the Higgs sector. In our analysis we used the most up to date low energy experimental data, including the CCFR κ^2 . Thanks to these we obtained new limits on left-right symmetric model parameters that are comparable with those from high energy physics.

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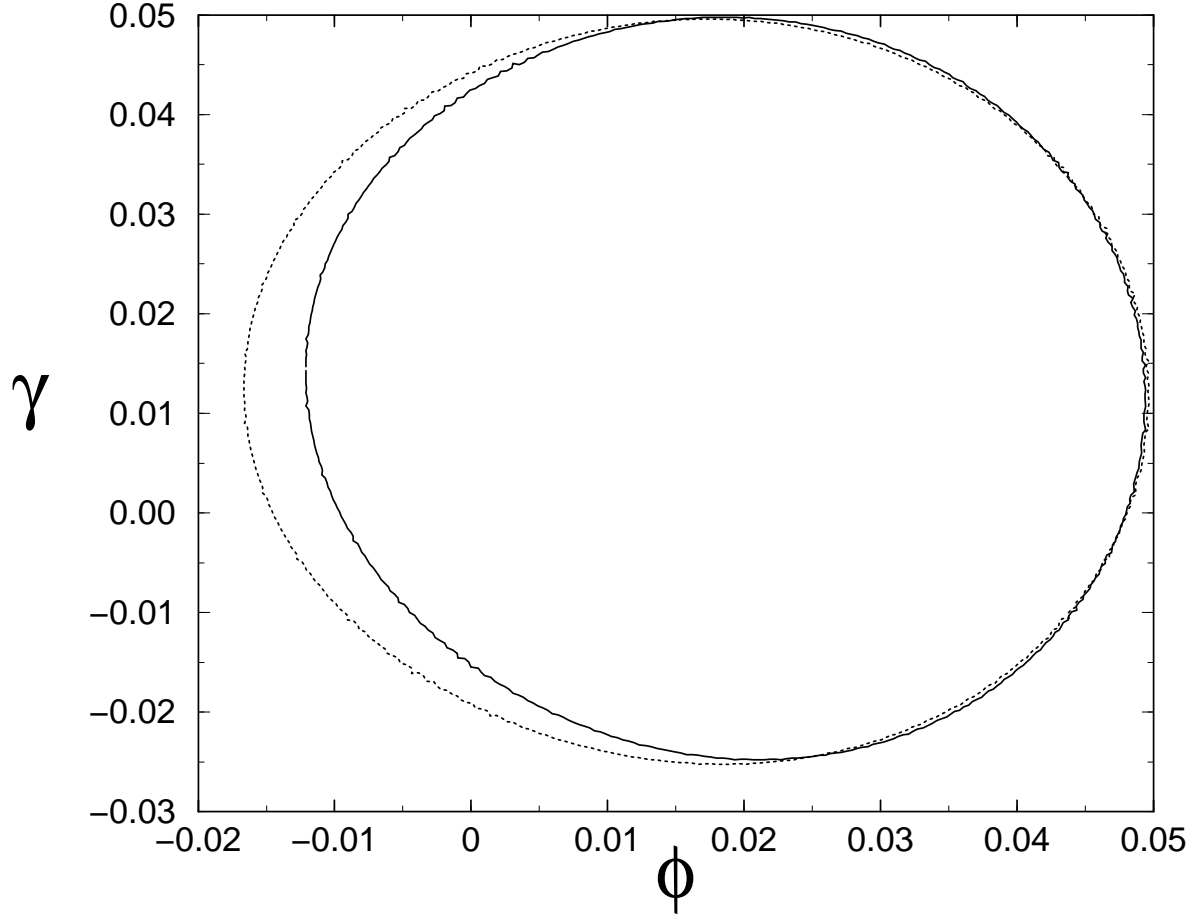


Fig. 1. 90 % C.L. region for allowed $\gamma - \phi$ parameters. Dashed line describes result when data of Table 1 are taken into account. Solid line takes into account additional data given in Eq.(47).

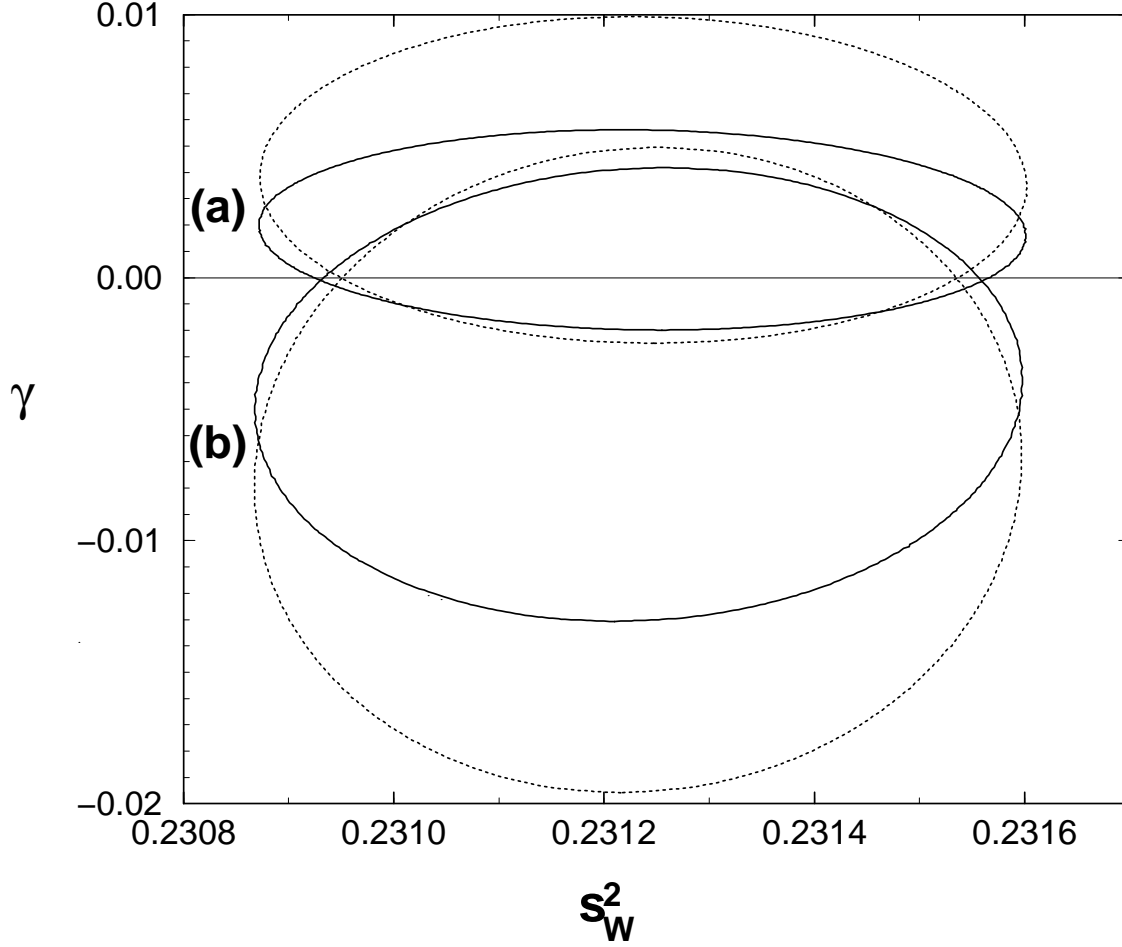


Fig. 2. 90 % C.L. region for the allowed $\gamma - \sin^2 \Theta_W$ parameters when relations Eqs.(17-21) are taken into account. Two upper ellipses (a) realize models with $\epsilon = 1$ (possible $W_L - W_R$ mixing). Two lower ellipses (b) give results for $\epsilon = 0$ (no $W_L - W_R$ mixing). The dashed line describes the result when the data of Table 1 is taken into account. The solid line takes into account the additional data given in Eq.(47).